Full-Stack Quantum Computing

States

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \qquad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \qquad |-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$|i\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle \qquad |-i\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

1-Qubit Gates

$$X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \qquad Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix} \qquad Y = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix} \qquad H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$$

2-Qubit Gates

$$CNOT = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\\0 & 1 & 0 & 0\end{bmatrix}$$

Assuming $|a\rangle$ and $|b\rangle$ are in the $|0\rangle$ and $|1\rangle$ basis:

$$CNOT(|b\rangle \otimes |a\rangle) = |b \oplus a\rangle \otimes |a\rangle$$

Identities

$$X|0\rangle = |1\rangle \quad X|1\rangle = |0\rangle$$
$$H|0\rangle = |+\rangle \quad H|1\rangle = |-\rangle$$
$$H|+\rangle = |0\rangle \quad H|-\rangle = |1\rangle$$
$$X = HZH \quad Z = HXH$$